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IMPLEMENTATION OF THE MAXIMUM SUSTAINABLE YIELD UNDER AN AGE-STRUCTURED MODEL[1]

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Abstract: One of the main goals stated in the proposals for the Common Fisheries Policy (CFP) reform is achieving maximum sustainable yield (MSY) for all European fisheries by 2015. In this paper, we consider the mechanism design problem for allocation of fishing rights to achieve MSY harvesting conditions. We study an age-structured fish population model and apply this model for a fishing environment including two fishermen having perfect or imperfect fishing selectivity. If we assume that fishermen are non-satiated and they fulfill their remaining quotas through capturing untargeted (less revenue-generating) fish after targeted fish population is fully caught, the fix ratio of the catch of targeted fish to untargeted fish, derived from catchability coefficients, is not valid anymore. As a result, we show that not only the age-structure or fishing technology but also the estimated level of MSY is steering the optimal allocation of quotas. Accordingly, we determine technology-based optimal quota shares for each fisherman at particular MSY levels. We also show that the optimal allocation of fishing quotas does not have a bang-bang nature under imperfect fishing selectivity.

Keywords: Age-structured model, Allocation of quotas, Fishing technology, Maximum sustainable yield, Mechanism design, Rights-based management, Total allowable catch

Abbreviations: Common Fisheries Policy (CFP), Maximum sustainable Yield (MSY), Rights-based management (RBM), Total allowable catch (TAC).

JEL Classifications: D45, D47, Q22, Q56, Q58

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1 Introduction

In European fisheries, maximum sustainable yield (MSY) has not been achieved for all economically valuable fish stocks. According to Facts and Figures on the Common Fisheries Policy (2012), only 11 fish stocks in the Atlantic shoreline and 21 fish stocks in the Mediterranean are fished at MSY. Most of the other fish stocks remain outside safe biological limits and are overfished. This implies that the provision of sustainable fish stock levels, which is one of the most important environmental objectives of the Common Fisheries Policy (CFP), have not yet been achieved in European fisheries. There is a consensus in the European Union (EU) on the medium term benefits of implementing MSY on environmental, social and economic sustainability. Therefore, achievement of MSY for all fish stocks by 2015 has become prominent as one of the main topics within the scope of the CFP reform proposals. However, it is not easy to put the concept of MSY into practice, thus the goal of MSY has not been accomplished for more than 30 years in European waters.

These discussions boil down to a question of how MSY can be sustainably implemented for a fish stock. Management systems play the key role in sustainably implementation of MSY. Fisheries in the EU are managed through various systems. The most prominent options among those are rights-based management (RBM) systems. In this paper, we analyze the implementation of MSY approach under the individual quota system, which is one of the most well-known types of RBM systems. The main purpose of the paper is to describe an effective design for quota allocation mechanisms that guarantees sustainability of fish stocks and increases market efficiency.

Efficiency of an individual quota system depends on three main steps of the system. The initial step is the accurate determination of the total allowable catch (TAC) level. The second step is the implementation of a well-designed quota allocation mechanism, and the final step is the efficient control of the output landed by fishermen. This study combines the first and second steps stated above. The connection of the first two steps arises from one of our main results which indicates that TAC level has to be equal to MSY level of fishing mortalities in determination of the optimal quota shares. The paper considers the mechanism design problem from the viewpoint of a social planner. Firstly, we know that precise data about the structure of a given fish population is required to manage the fish stock in accordance with the MSY objectives. Given that MSY is calculated for a given fish stock, we present an RBM mechanism implementing the outcome of MSY fishing mortalities in a simple age-structured fish population model with three interacting age classes of a single fish stock, and without loss of generality with two fishing agents having perfect or imperfect fishing selectivity. The analysis basically indicates that a well-designed RBM system is required to align the interest of all agents in the fishing industry to implement MSY. Within this framework, we determine fishing technology-based optimal quota levels at particular MSY levels. Basically, we demonstrate that the determination of the optimal quota shares is not independent of the specified TAC levels. As a result, we showed that not only the age-structure and fishing technology but also the estimated level of MSY is steering the optimal allocation of quota shares obtained by each fisherman.

There is a vast literature on age-structured fish population models. In recent years, Clark (2010), Tahvonen (2009a, 2009b, 2010), Quaas et al. (2010) and Skonhoft et al. (2012), among others, have contributed to the literature of age-structured modeling for fisheries. Moreover, Armstrong (1999) investigated the harvest shares of trawlers and coastal vessels at particular TAC levels using the actual allocation rule for the Norwegian cod fishery. See also Armstrong and Sumaila (2001), Björndal and Brasao (2006), Stage (2006), and Diekert et al. (2010) for more on applications of age-structured model to different case studies. Skonhoft et al. (2012) have recently formulated an age-structured model and derived MSY fishing mortalities similar to that of Reed's (1980). They estimate optimal fishing efforts that maximize the total profits of the two agents targeting the young mature fish and the old mature fish, respectively. However, the implementation of MSY approach under an individual quota system has not been discussed adequately. In most of the studies using age-structured fish population models, catchability coefficients are used as the key parameters for estimation of fishermen's catch compositions. Skonhoft et al. (2012) also used fixed catchability coefficients to estimate fishermen's catch compositions and optimal fishing efforts. In this paper, the age-structured fish population model developed by Skonhoft et al. (2012) is employed and fishing mortality rates at MSY are calculated using a simple Lagrangian method proposed by Skonhoft et al. (2012). Additionally, we consider an extra condition for catch composition of fishermen. Depending on the cost structures, after the high-revenue generating (old mature) fish are fully harvested, fishermen may act in a non-satiated behavior such that they may prefer to maintain their operations only for catching less-revenue generating (young mature) fish. This kind of decision depends on the cost structure of fishing operations. In this study, we investigate the mechanism design problem for allocation of fishing quotas by considering this non-satiated behavior of fishermen. In this sense, under the condition that fishermen fulfill their remaining quotas through capturing untargeted fish after the targeted fish population is fully harvested, the fix ratio of the catch of targeted fish to untargeted fish derived by the catchability coefficients is no longer valid. The reason why we consider such a case is that we aim to indicate the results of the non-satiated behavior of fishermen on the implementation of the MSY. In this environment, we work on three cases showing differences in fishing technologies of fishermen, and we determine technology-based optimal quota shares for particular MSY levels. Note that the maximum sustainable biomass yield harvesting strategies are described under different assumptions in the literature. Our main contribution is to show how to implement the optimal harvesting policy under different assumptions about fishing structures.

The rest of the paper is organized as follows. In the next section, we introduce the model and provide basic definitions. In Section 3, we formulate the optimization problem to find MSY fishing mortalities. Section 4 studies implementation of MSY fishing levels. Section 5 provides a numerical illustration of our main results. Section 6 discusses policy implications of our analysis and contains concluding remarks.

2 The Model

2.1 Population Model

The population model is based on three cohorts of the fish population. The juveniles are the members of the youngest class in the population. They are neither harvestable nor members of the spawning stock, while the old mature and young mature fish are both harvestable and members of the spawning stock. In addition, the old mature fish has higher fertility rate than the young mature fish has, as supposed by Reed (1980). Moreover, weight per fish is higher for the older fish ($w_0 < w_1 < w_2$). It is considered that the juvenile has no market value, whereas price per weight for the old mature fish is higher than the price per weight for the young mature fish ($p_0 = 0,\ p_1 < p_2$). Owing to the fact that weight per fish and price per weight are less for the young mature fish than the old mature fish, the young mature fish refers to less revenue-generating fish in our analysis. The population at any time t is defined as follows:

Juveniles, $X_{0,t}$ (age < 1),

Young matures, $X_{1,t}$ (1 ≤ age < 2),

Old matures, $X_{2,t}$ (2 ≤ age).

In the model, we employ the Beverton-Holt type of recruitment function, which is increasing and concave for both age classes (Beverton and Holt, 1957). The number of recruits to the fish population at time t is:

$$X_{0,t} = R\left(X_{1,t}, X_{2,t}\right) = a\left(X_{1,t} + \beta X_{2,t}\right)/[b + (X_{1,t} + \beta X_{2,t})]. \quad (1)$$

The number of recruits depends on the old mature and young mature fish populations and parameters of a, b and β. The parameters of a and b are the scaling and shape parameters, respectively. Besides, β is the fertility parameter indicating the higher natural fertility of the old mature fish than of the young mature fish. The numbers of the juveniles and young mature fish at time t+1 are defined by the following equations:

$$X_{0,t+1} = R(X_{1,t+1}, X_{2,t+1}), \quad (2)$$

and

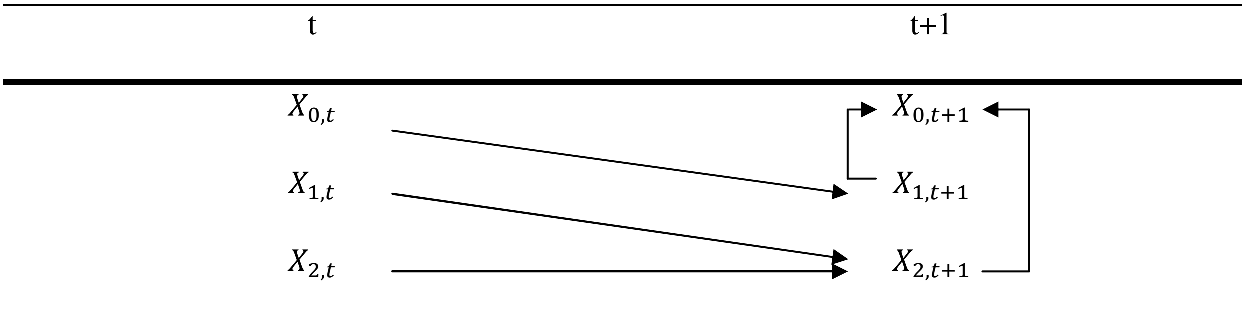
$$X_{1,t+1} = s_0\, X_{0,t} = s_0\, R\left(X_{1,t}, X_{2,t}\right). \quad (3)$$

The number of the old mature fish at t+1 is given as:

$$X_{2,t+1} = s_1\,(1 - f_{1,t})\, X_{1,t} + s_2\,(1 - f_{2,t})\, X_{2,t}. \quad (4)$$

In the above notation, s_0 is the fixed natural survival rate of the juveniles. Moreover, $f_{1,t}, s_1$ and $f_{2,t}, s_2$ are the fishing mortality rates and fixed natural survival rates of the young mature and old mature fish, respectively. Life cycle scheme for this age-structured fish population is depicted in Table 1. The arrows directed to the right column from the left one show the ageing structure from t to t+1, and the arrows within the right column directed to the upper cells show the recruitment structure of the fish population.

Table 1 Life cycle scheme of an age-structured fish population



In this study, it is considered that fishing activity occurs after spawning and before natural mortality. We aim to describe the quota allocation mechanism at the population equilibrium maximizing the population growth. Hence, fixed fishing mortalities at the population equilibrium (steady-state outcomes) are taken into consideration ($X_{i,t+1} = X_{i,t} = X_i$). We also assume that the total biomass of the old mature fish is less than the total biomass of the young mature fish ($w_2\, X_2 < w_1\, X_1$). This assumption refers to a stylized real life situation but it is also easy to extend our analysis to cover all possible cases. The following equations are the constraints of our maximization problem. Eq. (3') represents the spawning constraint and Eq. (4') represents the recruitment constraint.

$$X_1 = s_0\, R(X_1, X_2), \tag{3'}$$

$$X_2 = s_1\,(1 - f_1)\, X_1 + s_2\,(1 - f_2) X_2. \tag{4'}$$

The population model developed by Skonhoft et al. (2012) is described so far. Additionally, we need the condition stated above, the total biomass of the old mature fish is less than the total biomass of the young mature fish at the population equilibrium, to simplify our analysis. In what follows, under given age-structured population dynamics, catch compositions of fishermen are described and optimal allocation of quotas at MSY level of fishing is investigated for different cases.

2.2 Catch Compositions

In this mechanism design problem, we consider two fishermen characterized by their fishing technologies. Fishing technologies of fisherman 1 and fisherman 2 are denoted as j_1 and j_2, respectively. Our analysis focuses on the situation in which both fishermen target the old mature class. Table 2 summarizes the three cases to be carried out. Case 1 refers to the fishing environment including two fishermen having perfect fishing selectivity. In Case 1, since both fishermen have perfect fishing selectivity it is easy to see that the quota allocation does not play a role in the achievement of MSY conditions. However, MSY level of fishing, which is equal to the TAC level, should be known to state whether MSY conditions can be achieved or not under this fishing environment. In Case 2, which is a more complex case, we investigate the optimal harvesting policy in a fishing environment including two fishermen having perfect and imperfect fishing selectivity, respectively. On the other hand, there are two fishermen having imperfect fishing selectivity in Case 3. For these three cases, we implement MSY harvesting policy outcomes given the fact that both fishermen target the old mature fish. The mechanism described in this paper can also be used to find optimal harvesting policy for a fishery including fishermen targeting different groups of fish.

Table 2 Cases on Catch Composition of Fishermen

	Fishing technology level of fisherman 1	Fishing technology level of fisherman 2
Case 1	$j_1 = 1$ (No bycatch)	$j_2 = 1$ (No bycatch)
Case 2[4]	$j_1 = 1$ (No bycatch)	$0.5 < j_2 < 1$ (Bycatch)
Case 3	$0.5 < j_1 < 1$ (Bycatch)	$0.5 < j_2 < 1$ (Bycatch)

Technology level of j_i simply derived from the catchability coefficients of fisherman i. Catchability coefficients of the old mature fish (targeted fish) and young mature fish (untargeted fish) for fisherman i are denoted by q_i^2 and q_i^1, respectively. Under given catchability coefficients, we can simply write fishing technology of fisherman i as $j_i = q_i^2 / q_i^1 + q_i^2$. At a given j_i level, catch of targeted fish of fisherman i is equal to $j_i x 100$ percent of the total catch of that fisherman. The lower bound for fishing technologies are taken as 0.5 because we consider that both fishermen have fishing technologies compatible with capturing the targeted fish. Therefore, j_1 and j_2 are always greater than 0.5. For instance, given that $j_i = 0.8$, then it means that fisherman i captures $n/4$ tonnes of young mature fish while capturing n tonnes of the old mature fish. In the estimation of fishing technology, we do not consider the catch of the juveniles or other type of marine species, which has no market value and hence are not landed by fishermen. Fishermen, fulfill the quotas assigned to them through capturing only targeted fish (under perfect fishing selectivity) or both targeted and untargeted but revenue-generating fish (under imperfect fishing selectivity). Therefore, fishing technology of a fisherman is just related to catchability of the old mature and young mature fish (bycatchability) of that fisherman.

[4] The case in which $0.5 < j_1 < 1$ and $j_2 = 1$ is omitted since it is the symmetric of Case 2.

To implement the MSY level, we first need to set the TAC equal to the MSY fishing mortalities. Since catching at the MSY level fixes the total biomass to be harvested in a given period, the population equilibrium level will be sustained. We investigate the optimal individual quota levels for particular MSY (=TAC) levels. The main task then becomes how to determine individual fishing quotas for particular TAC levels.

The quota share assigned to fisherman i, $0 \leq \alpha_i \leq 1$, is closely related to his fishing technology which is derived from catchability coefficients. However, the ratio between catch of the old mature and young mature fish of a fisherman is valid till the old mature fish population is fully harvested. Hence, catch compositions cannot always be estimated by using the ratio derived from catchability coefficients.

We assume that until one of the fishermen's quota is exhausted, fishermen harvest the same total weight of fish at a given time duration. That is, fishermen harvest the fish population in weight increments due to identical capacity of fishing vessels. They divide the weight increments equally until one of the fishermen's quota is reached. After one of the fishermen's quota is exhausted, the other fisherman fulfills his quota with the old mature fish (under perfect fishing selectivity) or the old mature and young mature fish (under imperfect fishing selectivitiy) if all old mature fish population had not been completely harvested. Otherwise, the fisherman fulfills his quota with only the young mature fish (under imperfect fishing selectivity). For example, in Case 2, if fisherman 1 harvests n tonnes of old mature fish at a given time, then fisherman 2 harvests $j_2 n$ tonnes of old mature fish and $(1 - j_2)n$ tonnes of young mature fish given that $n + j_2 n \leq w_2 X_2$ and n is less than the quota assigned to both fisherman 1 and fisherman 2, $n \leq \min\{\alpha_1 TAC, \alpha_2 TAC\}$. If $n = \alpha_1 TAC < \alpha_2 TAC$ and $w_2 X_2 = n + j_2 n$, then fisherman 2 fulfills his remaining quota with young mature fish and hence harvests $\alpha_2 TAC - n$ additional tonnes of young mature fish. On the other hand, if $n = \alpha_1 TAC < \alpha_2 TAC$ and $w_2 X_2 - n > j_2 n$, then fishermen 2 fulfills his remaining quota with both age classes according to his technological fishing constraint and the assigned quota level. If $n = \alpha_2 TAC < \alpha_1 TAC$, then fisherman 1 fulfills his remaining quota with old mature fish and hence harvests $\alpha_1 TAC - n = w_2 X_2 - (1 + j_2)n$ additional tonnes of old mature fish. For example, in Case 3, if fisherman 1 harvests $j_1 n$ tonnes of old mature fish and $(1 - j_1)n$ tonnes of young mature fish at a given time, then fisherman 2 harvests $j_2 n$ tonnes of old mature fish and $(1 - j_2)n$ tonnes of young mature fish given that $n \leq \min\{\alpha_1 TAC, \alpha_2 TAC\}$. If without loss of generality $n = \alpha_1 TAC < \alpha_2 TAC$, then fisherman 2 fulfills his quota according to his technological fishing constraint, the assigned quota level and the composition of surviving fish as in Case 2.

The following example is provided to clarify this situation. First of all, suppose that all quotas are assigned to fisherman 1 who has an imperfect fishing selectivity and TAC is determined at a level such that $TAC \leq w_2 X_2 + [(1 - j_1)/j_1]\, w_2 X_2$. In this case, fisherman 1 captures n tonnes of targeted fish where $n = j_1 TAC \leq w_2 X_2$, and $[(1 - j_1)/j_1]\, n$ tonnes of untargeted fish. The ratio derived from the catchability coefficients is valid in this case. On the other hand, if TAC is determined above $w_2 X_2 + [(1 - j_1)/j_1]\, w_2 X_2$, which is the cut-off level, then catch of the young mature fish of fisherman i will exceed its expected level derived by the catchability coefficients. The reason is that, after the old mature fish population is fully harvested, fisherman 1 will fulfill his remaining quota through capturing the young mature fish under the assumption that fishermen are non-satiated. The weight of catch of targeted fish by fisherman 1 will be equal to $w_2 X_2$ and the rest of his catches will consist of $TAC - w_2 X_2$ weight of the young mature fish. As a result, given that fisherman 1 targets the old mature fish and bycatches the young mature fish, the ratio of the weight of catch of targeted fish to the weight of bycatch derived by the catchability coefficients is provided until the old mature fish population is fully harvested. Additionally, in our analysis we did not take into consideration the TAC levels, which are higher than or equal to the total biomass of the young mature fish.

Thus, in our analysis $TAC < w_1 X_1$ is always provided as a realistic situation. We continue by defining catch compositions of fishermen at different TAC levels.

The initial process of estimating the catch compositions of fishermen is to determine the cut-off levels for TAC under giving fishing technology conditions. For example, consider a fishing environment including two fishermen having imperfect fishing selectivity ($0.5 < j_1 < 1,\ 0.5 < j_2 < 1$). Under given fishing technologies, the cut-off levels for TAC are $w_2 X_2 + [(1 - j_1)/j_1]\, w_2 X_2$ and $w_2 X_2 + [(1 - j_2)/j_2]\, w_2 X_2$. If MSY is calculated below the minimum of these cut-off levels such that $TAC < \min\{ w_2 X_2 + [(1 - j_1)/j_1]\, w_2 X_2, w_2 X_2 + [(1 - j_2)/j_2]\, w_2 X_2\}$, then catch composition of fisherman i can be defined as follows:

$$\alpha_i\, TAC = h_2^i\, w_2 X_2 + b_1^i\, w_1 X_1,\ \textstyle\sum_{i=1,2} \alpha_i = 1 \tag{5}$$

where α_i is the quota share assigned to fisherman i as a percentage of TAC. Then, total harvest of fisherman 1 is equal to $\alpha_1\, TAC$ consisting of $h_2^1\, w_2 X_2$ tonnes of old mature (targeted) fish and $b_1^1\, w_1 X_1$ tonnes of young mature (untargeted) fish. Likewise, total harvest of fisherman 2 consists of $h_2^2\, w_2 X_2$ tonnes of old mature fish and $b_1^2\, w_1 X_1$ tonnes of young mature fish. Catch compositions of fishermen for the specified TAC level can also be expressed in the following way:

$$h_2^1\ w_2 X_2 = j_1\, \alpha_1\, TAC > b_1^1\ w_1 X_1 = (1 - j_1)\, \alpha_1\, TAC,$$
$$h_2^2\, w_2 X_2 = j_2\, \alpha_2\, TAC > b_1^2\, w_1 X_1 = (1 - j_2)\, \alpha_2\, TAC.$$

This simple pattern is given to show how our methodology works under described fishing environment for TAC levels such that $TAC < \min\{ w_2 X_2 + [(1 - j_1)/j_1]\, w_2 X_2,$

$w_2\, X_2 + [(1 - j_2)/j_2]\ w_2\, X_2\}$. In this environment, the weight of target fish catch of fisherman i is higher than the weight of bycatch of that fisherman. As a result, we obtain the inequalities of $h_2^1\, w_2\, X_2 > b_1^1 w_1\, X_1$ and $h_2^2\, w_2\, X_2 > b_1^2\, w_1\, X_1$ for the given conditions.

On the other hand, TAC may also be set at levels between the two cut-off levels or higher than the maximum of the cut-off levels. In the light of the preceding discussions, it is easy to see that catch compositions of fishermen show differences according to the TAC levels and catch ratio is not fixed for the later cases. Within this framework, we investigate the optimal quota allocations at particular TAC levels under three possible cases for catch compositions of the fishery. In the next section, we present the robust methodology to derive MSY fishing levels for this environment.

3 Maximum Sustainable Yield

In this part of the study, MSY level is investigated under the age-structured fish population model. The problem of finding the MSY harvesting strategies for this environment has been studied in the literature. We present this section for the completeness of the paper. We directly apply the approach of Skonhoft et al. (2012) to show the optimal fishing mortality conditions at MSY. The harvest function is:

$$Y = \ f_1\, w_1\, X_1 + \ f_2\, w_2\, X_2. \tag{6}$$

The constraints for the maximization problem are:

$$X_1 = s_0\ \ R(X_1, X_2)\,, \tag{3'}$$

and

$$X_2 = s_1\ (1 - f_1)\ X_1 + s_2\,(1 - f_2) X_2. \qquad (4')$$

The Lagrangian function and the first order necessary conditions, which are also sufficient for this problem, derived from the simple Lagrangian model are:

$$L = f_1\, w_1\, X_1 + f_2\, w_2\, X_2 - \varphi\,[X_1 - s_0\, R(X_1, X_2)] - \mu\,[X_2 - s_1\,(1 - f_1)\, X_1 + s_2\,(1 - f_2)\, X_2] \qquad (7)$$

$$\partial L/\partial f_1 = (w_1 - \mu\, s_1)\, X_1 \leq 0;\ 0 \leq f_1 < 1\,, \qquad (8)$$

$$\partial L/\partial f_2 = (w_2 - \mu\, s_2)\, X_2 \leq, \geq 0;\ 0 \leq f_2 \leq 1, \qquad (9)$$

$$\partial L/\partial X_1 = f_1\, w_1 + \varphi\left(s_0\, R_1^{'} - 1\right) + \mu\, s_1\,(1 - f_1) = 0, \qquad (10)$$

$$\partial L/\partial X_2 = f_2\, w_2 + \varphi s_0\, R_2^{'} + \mu\, s_2\,[(1 - f_2) - 1] = 0. \qquad (11)$$

It can be extracted from the first order necessary conditions that $\partial L/\partial f_1$ and $\partial L/\partial f_2$ are independent of the recruitment function. It is also assumed that the natural survival rates of the old mature and young mature fish do not differ at a significant level. Hence, the ratios of weights to natural survival rates satisfy the inequality of $w_2/s_2 > w_1/s_1$ (Skonhoft et al., 2012). The conditions for fishing mortalities, which can be derived from the first order necessary conditions are:

1) Given that $\mu = w_1/s_1 < w_2/s_2$, then $\partial L/\partial f_1 = 0$ and $\partial L/\partial f_2 > 0$. A one-unit increase in f_1 does not change the value of the objective function. However, a one-unit increase in f_2 increases the value of the objective function at an amount of $(w_2 - \mu\, s_2)\, X_2$. Hence, f_2 should be maximized and f_1 should be such that $0 < f_1 < 1$. Therefore, we can write the fishing mortality conditions as $0 < f_1 < 1$ and $f_2 = 1$.

2) Given that $w_1/s_1 < \mu < w_2/s_2$, then $\partial L/\partial f_1 < 0$ and $\partial L/\partial f_2 > 0$. Hence, f_2 should be maximized and f_1 should be minimized to maximize the objective function. This implies that f_1 and f_2 should be 0 and 1, respectively.

3) Given that $w_1/s_1 < \mu = w_2/s_2$, then $\partial L/\partial f_1 < 0$ and $\partial L/\partial f_2 = 0$. Hence, f_2 should be such that $0 < f_2 < 1$ and f_1 should be minimized, which is satisfied at $f_1 = 0$.

4) Given that $w_1/s_1 < w_2/s_2 < \mu$, then $\partial L/\partial f_1 < 0$ and $\partial L/\partial f_2 < 0$. Under this condition, f_2 and f_1 should be minimized. This solution is not economically sustainable since the total equilibrium biomass harvested cannot be equal to zero.

5) Given that $\mu < w_1/s_1 < w_2/s_2$, then $\partial L/\partial f_1 > 0$ and $\partial L/\partial f_2 > 0$. Under this condition, f_2 and f_1 should be maximized. This one also cannot be an optimal allocation since fishing mortality of the young mature fish should be less than 1 ($f_1 < 1$) to provide the sustainability of the fish population.

Table 3 depicts the fishing mortality rates at MSY under different conditions for $\mu, w_1/s_1$ and w_2/s_2. The 4th and 5th results cannot be the optimal solutions as discussed above. Given that the shadow value of the spawning constraint satisfies the inequalities 4 and 5, MSY is not achieved. Therefore, the shadow value of the spawning constraint must satisfy the inequalities of 1, 2 or 3 to achieve MSY.

Table 3 Fishing mortality rates at MSY

		$\partial L/\partial f_1$	$\partial L/\partial f_2$	f_1	f_2
1)	$\mu = w_1/s_1 < w_2/s_2$	$= 0$	> 0	$0 < f_1 < 1$	$= 1$
2)	$w_1/s_1 < \mu < w_2/s_2$	< 0	> 0	$= 0$	$= 1$
3)	$w_1/s_1 < \mu = w_2/s_2$	< 0	$= 0$	$= 0$	$0 < f_2 < 1$

We find the fishing mortality rates through the maximization problem. As mentioned before, TAC is set at MSY, which is equal to total fishing mortalities. The following equation defines the TAC level for our allocation mechanism:

$$TAC = \text{MSY} = f_1\, w_1\, X_1 + f_2\, w_2\, X_2. \qquad (12)$$

It should be noted that in every case, we only analyze the fishing mortality solutions, which are compatible with the fishing technologies given in that case. For instance, given that both fishermen have imperfect fishing selectivity, then we do not take into consideration the fishing mortality solutions such as $f_1 = 0$ and $f_2 = 1$ or $f_1 = 0$ and $f_2 < 1$ because achievement of MSY at $f_1 = 0$ is not possible since both fishermen bycatch at given fishing technologies. Hence, we look for optimal allocation of quotas at fishing mortalities such that $0 < f_1 < 1$ and $f_2 = 1$, for the given case. Briefly, we determine the optimal quota levels at fishing mortalities, which can be obtained under given fishing technologies. Thus, rather than representing this kind of impossible solutions in the summary tables, we prefer to eliminate these solutions initially and hence they are not represented in the tables showing the optimal quota levels. In addition, as a result of our assumption considering the non-satiated behavior of fishermen, both fishermen always try to fulfill the quotas assigned to them. However, there is a situation that may result in quota waste despite the fact that fishermen are non-satiated. If the TAC is set above the cut-off points and quotas are not efficiently allocated, then the fisherman who has perfect fishing selectivity (in Case 1 or Case 2) may waste a part of the quotas assigned to him. This situation is observed if there are still remaining quotas of the fisherman who has perfect selectivity after the old mature fish population is fully caught. Thus, a part of the quota is wasted since that fisherman has perfect fishing selectivity and cannot fulfill his remaining quotas through capturing young mature fish. However, we consider that the social planner is aware of the possibility and eliminates these types of allocations, which result in waste of quota. That is, for the optimal quota levels of α_i, $\alpha_i TAC = h_2^i\, w_2\, X_2$ if j_i =1 for all $i \in \{1,2\}$.

4 Implementation of the Maximum Sustainable Yield

In this section, we start to work on our main problem, implementation of MSY under different assumptions about fishing technologies. Let us begin our analysis on optimal allocation of quotas at particular MSY levels for Case 1.

***Case 1**: Suppose that both fishermen have perfect fishing selectivity ($j_1 = j_2 = 1$).*

Case 1.1: If the optimal fishing mortalities of the young mature and old mature fish are found as $f_1 = 0$ and $0 < f_2 < 1$, then TAC is determined at a level such that $TAC = f_2\, w_2\, X_2$ by Eq. 12. Since both fishermen have perfect fishing selectivity, MSY is achieved regardless of the allocation of quotas which means that the total weight of catches will be equal to $f_2\, w_2\, X_2$ at every combination of quota shares.

Case 1.2: If the optimal fishing mortalities of the young mature and old mature fish are found as $f_1 = 0$ and $f_2 = 1$, then TAC is equal to $w_2\, X_2$. Under these conditions, MSY is achieved regardless of quota allocation as observed in Case 1.1. Hence, total weight of catches will always be equal to $w_2\, X_2$.

Optimal allocation of quotas for Case 1 is summarized in Table 4.

Table 4 Quota allocation mechanism for Case 1

<table>
<tr><td rowspan="5">Case 1

$j_1 = 1$,
$j_2 = 1$</td><td>The discounted biomass conditions</td><td>Fishing mortality rates at MSY</td><td>Optimal allocation of quotas at MSY</td></tr>
<tr><td colspan="3">i. $TAC = f_2\, w_2\, X_2 < w_2\, X_2$</td></tr>
<tr><td>$w_1/s_1 < \mu = w_2/s_2$</td><td>$f_1 = 0, 0 < f_2 < 1$</td><td>$\{\alpha \mid \alpha_i \epsilon [0,1] \wedge \alpha_1 + \alpha_2 = 1\}$</td></tr>
<tr><td colspan="3">ii. $w_2\, X_2 \leq TAC = f_1\, w_1\, X_1 + w_2\, X_2$</td></tr>
<tr><td>$w_1/s_1 < \mu < w_2/s_2$</td><td>$f_1 = 0,\ f_2 = 1$</td><td>$\{\alpha \mid \alpha_i \epsilon [0,1] \wedge \alpha_1 + \alpha_2 = 1\}$</td></tr>
</table>

Result 1: Given that both fishermen have perfect fishing selectivity, MSY will be achieved for every combinations of individual quota shares, α_1 *and* α_2 *where* $\alpha_1 + \alpha_2 = 1$, *under two different conditions. Firstly, if the optimal fishing mortalities of the young mature and old mature fish are found as* $f_1 = 0$ *and* $0 < f_2 < 1$ *and TAC is set at* $f_2\, w_2\, X_2$ *and secondly, if the optimal fishing mortalities of the young mature and old mature fish are found as* $f_1 = 0$ *and* $f_2 = 1$ *and TAC is set at* $w_2\, X_2$*, then MSY is achieved. Otherwise, MSY is not achieved.*

The first case investigates the optimal quota levels for fishermen having perfect fishing selectivity. The second case, which is a more complex one, studies the optimal allocation of quotas for fishermen having different fishing technologies.

Case 2*: Suppose that fisherman 1 has perfect fishing selectivity and fisherman 2 has imperfect fishing selectivity (*$j_1 = 1$ *and* $0.5 < j_2 < 1$*).*

Case 2.1: If the optimal fishing mortalities of the young mature and old mature fish are found as $f_1 = 0$ and $0 < f_2 < 1$, then TAC is determined at a level satisfying $TAC = f_2\, w_2\, X_2$. Under these conditions, MSY is achieved if all quotas are assigned to fisherman 1 since fishing mortality of the young mature fish can only be equal to zero under the quota allocation of $\alpha_1 = 1$ and $\alpha_2 = 0$. Hence, MSY is achieved only at the corner solution: $\alpha_1 = 1$ and $\alpha_2 = 0$.

Case 2.2.a: If the optimal fishing mortalities of the young mature and old mature fish are found as $f_1 = 0$ and $f_2 = 1$, then TAC is equal to $w_2\, X_2$. Owing to the fact that fishing mortality of the young mature fish can only be equal to zero at $\alpha_1 = 1$, $\alpha_2 = 0$, MSY will be achieved if all quotas are assigned to fisherman 1.

Case 2.2.b: If the optimal fishing mortalities are found at some levels such that $0 < f_1 < 1$ and $f_2 = 1$ where $f_1\, w_1\, X_1 < [(1 - j_2)/j_2]\, w_2\, X_2$, then TAC is set at a level such that $w_2\, X_2 < TAC = f_1\, w_1\, X_1 + w_2\, X_2 < [(1 - j_2)/j_2]\, w_2\, X_2 + w_2\, X_2$. If all quotas are assigned to fisherman 1, it is easy to see that MSY cannot be implemented since fisherman 1 does not bycatch. On the other hand, if all quotas are assigned to fisherman 2, then according to his fishing technology level he fulfills his total quotas before capturing all old mature fish. This implies that his total harvest consists of old mature fish less than $w_2\, X_2$ and young mature fish more than $f_1\, w_1\, X_1$. However, to achieve MSY, total weight of bycatch of fisherman 2 should be equal to $f_1\, w_1\, X_1$, and total weight of old mature fish should be equal to $w_2\, X_2$. Owing to the fact that both corner solutions are not optimum, we are looking for interior solutions. We now show that any quota shares such that $\alpha_1 \in [\underline{\alpha}_1, \overline{\alpha}_1]$ and $\alpha_2 = 1 - \alpha_1 > 0$ achieve MSY harvesting conditions for this case given that fishing mortality rate of young mature fish population is above a certain level. Suppose that $h_2^1\, w_2\, X_2 = n = \alpha_1 TAC$. Then, we have either $h_2^2\, w_2\, X_2 = w_2 X_2 - n \geq j_2 n$ or $h_2^2\, w_2 X_2 = w_2 X_2 - n < j_2 n$. Suppose that $h_2^2\, w_2 X_2 = w_2 X_2 - n \geq j_2 n$, and hence $f_1 w_1 X_1 \geq (1 - j_2)(w_2 X_2 - n)/j_2$. This implies that fisherman 1's quota exhausts first, and fisherman 2 fulfills his remaining quota. Moreover, $2\, w_2 X_2/(1 + j_2) \leq TAC$ since $\alpha_2 TAC \geq n$. Therefore, different quota shares produce the same TAC, which is equal to $f_1 w_1 X_1 + w_2 X_2$, if $f_1 \geq \frac{w_2 X_2 (1 - j_2)}{w_1 X_1 (1 + j_2)} = f^*$. At $\overline{\alpha}_1$, fisherman 2 should fulfill his quota by catching only young mature fish after all old mature fish are harvested. This implies that $h_2^2\, w_2\, X_2$ should be at minimum possible level such that $h_2^2\, w_2 X_2 = w_2 X_2 - n = j_2 n$ and $n = \overline{\alpha}_1\, TAC = w_2 X_2/(1 + j_2)$. Therefore, $\overline{\alpha}_1 = \frac{w_2 X_2}{(1 + j_2) TAC}$. At

$\underline{\alpha}_1$, fisherman 2 should harvest the maximum possible weight of old mature fish besides his young mature fish catch at the level of $f_1 w_1 X_1$. This is only possible if $\left(1-\underline{\alpha}_1\right) TAC\ (1-j_2) = f_1 w_1 X_1$. Thus, $\underline{\alpha}_1 = (w_2 X_2 - j_2 TAC)/[TAC(1-j_2)]$. Now, suppose that $h_2^2 w_2 X_2 = w_2 X_2 - n < j_2 n$. This is only possible if fisherman 2's quota depletes first and fisherman 1 fulfills his remaining quota with old mature fish. That is, $w_2 X_2 < TAC < 2 w_2 X_2/(1+j_2)$. Moreover, due to fisherman 1's technology, $b_1^2 w_1 X_1 = f_1 w_1 X_1 = [(1-j_2)/j_2]\ (w_2 X_2 - n) = (1-\alpha_1)\ TAC$. This implies that $\alpha_1 = (w_2 X_2 - j_2 TAC)/[(1-j_2) TAC]$.

Case 2.3: If the optimal fishing mortalities are found at some levels such that $0 < f_1 < 1$ and $f_2 = 1$ where $f_1\, w_1\, X_1 \geq [(1-j_2)/j_2]\, w_2\, X_2$, then TAC is set at a level satisfying $w_2\, X_2 + [(1-j_2)\ /\ j_2]\, w_2\, X_2 \leq TAC = f_1\, w_1\, X_1 + f_2\, w_2\, X_2$. Under these conditions, MSY is achieved if all of the quotas are assigned to fisherman 2 since fishermen 2 captures all of the old mature fish and also captures total weight of $f_1\, w_1\, X_1$ young mature fish by fulfilling the remaining quota with young mature fish after all old mature fish are harvested. On the other hand, if all quotas are assigned to fisherman 1, he wastes a part of his total quotas since he does not bycatch young mature fish. Hence, there is an upper bound of α_1, and fisherman 1 fulfills his all quotas by capturing old mature fish for all other quota shares that are equal to or below this α_1 level, $\overline{\alpha}_1$. We now show that any quota shares such that $\alpha_1 \epsilon [0, \overline{\alpha}_1]$ and $\alpha_2 = 1 - \alpha_1 > 0$ achieve MSY harvesting conditions for this case.

We look for possible interior solutions. Since TAC is set at a level equal to or higher than $w_2\, X_2 + [(1-j_2)\ /\ j_2]\, w_2\, X_2$, all old mature fish are captured regardless of the quota allocation. Moreover, fisherman 2 always fulfills his remaining quota with young mature fish for all feasible quota shares since $f_1\, w_1\, X_1 \geq [(1-j_2)/j_2]\, w_2\, X_2$. According to the fishing technology levels, we know that at any time period, if fisherman 1 captures n tonnes of the old mature fish, fisherman 2 captures $j_2\, n$ tonnes of the old mature fish and $(1-j_2)\, n$ tonnes of the young mature fish as long as $n + j_2 n \leq w_2 X_2$. At $\overline{\alpha}_1$, we have $h_2^1 = n =$

$\overline{\alpha}_1 TAC$, $h_2^2 = nj_2$ and $b_1^2 = f_1 w_1 X_1 \geq (1 - j_2)n$. Therefore, quota allocations satisfying $0 \leq \alpha_1 \leq w_2 X_2 / (1 + j_2)\, TAC$ will achieve MSY harvesting conditions.

Optimal allocation of quotas at different fishing mortalities for Case 2 is summarized in Table 5.

Table 5 Quota allocation mechanism for Case 2

	The discounted biomass conditions	Fishing mortality rates at MSY	Optimal allocation of quotas at MSY
Case 2	i. $TAC = f_2\, w_2\, X_2 \leq w_2\, X_2$		
	$w_1/s_1 < \mu = w_2/s_2$	$f_1 = 0$, $0 < f_2 < 1$	$\alpha_1 = 1, \alpha_2 = 0$
	ii. $w_2\, X_2 < TAC = f_1\, w_1\, X_1 + w_2\, X_2 < w_2\, X_2 + [(1 - j_2)/j_2]\, w_2\, X_2$		
	$w_1/s_1 < \mu < w_2/s_2$	$f_1 = 0$, $f_2 = 1$	$\alpha_1 = 1$, $\alpha_2 = 0$
$j_1 = 1$, $0.5 < j_2 < 1$	$\mu = w_1/s_1$, $\mu < w_2/s_2$	$f^* \leq f_1 < 1$, $f_2 = 1$	$\frac{[w_2\, X_2 - j_2 TAC\,]}{(1 - j_2)TAC} \leq \alpha_1 \leq \frac{w_2\, X_2}{(1 + j_2)TAC}$, $\alpha_2 = 1 - \alpha_1$
		$0 < f_1 < f^* < 1$, $f_2 = 1$	$\alpha_1 = \frac{[w_2\, X_2 - j_2 TAC\,]}{(1 - j_2)TAC}$, $\alpha_2 = 1 - \alpha_1$
	iii. $w_2\, X_2 + [(1 - j_2) / j_2]\, w_2\, X_2 \leq TAC = f_1\, w_1\, X_1 + w_2\, X_2$		
	$\mu = w_1/s_1 < w_2/s_2$	$0 < f_1 < 1$, $f_2 = 1$	$0 \leq \alpha_1 \leq w_2\, X_2 / (1 + j_2)\, TAC$, $\alpha_2 = 1 - \alpha_1$

In table 5, optimal quota shares are depicted for Case 2. To summarize the results, there are three sub-cases of Case 2. Firstly, if the optimal fishing mortalities are found such that $f_1 = 0$ and $0 < f_2 < 1$, then MSY is achieved if all of the fishing quotas are assigned to the fisherman who has perfect fishing selectivity, at a TAC level satisfying $\mathrm{TAC} = f_2\, \mathrm{w}_2\, \mathrm{X}_2 < \mathrm{w}_2\, \mathrm{X}_2$. Secondly, if the optimal fishing mortalities are found such that $0 < f_1 < 1$ and $f_2 = 1$, then MSY is achieved under the quota allocation of $\frac{[\mathrm{w}_2\,\mathrm{X}_2 - \mathrm{j}_2 \mathrm{TAC}]}{(1-\mathrm{j}_2)\mathrm{TAC}} \leq \alpha_1 \leq \frac{\mathrm{w}_2\,\mathrm{X}_2}{(1+\mathrm{j}_2)\mathrm{TAC}}$ and $\alpha_2 = 1 - \alpha_1$ if the TAC is set at a level satisfying $\frac{2\mathrm{w}_2\,\mathrm{X}_2}{1+j_2} \leq \mathrm{TAC} = f_1\, \mathrm{w}_1\, \mathrm{X}_1 + f_2\, \mathrm{w}_2\, \mathrm{X}_2 < \mathrm{w}_2\, \mathrm{X}_2 + [(1-\mathrm{j}_2)/\mathrm{j}_2]\, \mathrm{w}_2\, \mathrm{X}_2$. Moreover, MSY is achieved under the quota allocation of $\frac{[\mathrm{w}_2\,\mathrm{X}_2 - \mathrm{j}_2 \mathrm{TAC}]}{(1-\mathrm{j}_2)\mathrm{TAC}} = \alpha_1$ and $\alpha_2 = 1 - \alpha_1$ if the TAC is set such that $w_2 X_2 < TAC < \frac{2\mathrm{w}_2\,\mathrm{X}_2}{1+j_2}$. If the TAC is set at a level satisfying $\mathrm{w}_2\, \mathrm{X}_2 + [(1-\mathrm{j}_2)\,/\,\mathrm{j}_2]\, \mathrm{w}_2\, \mathrm{X}_2 \leq \mathrm{TAC}$, then optimal allocation of quotas is $0 \leq \alpha_1 \leq \mathrm{w}_2\, \mathrm{X}_2 /\, (1+\mathrm{j}_2)\, \mathrm{TAC}$ and $\alpha_2 = 1 - \alpha_1$. Finally, if the optimal fishing mortalities are found such that $f_1 = 0$ and $f_2 = 1$, then MSY is achieved if all of the fishing quotas are assigned to the fisherman who has perfect fishing selectivity, at a TAC level satisfying $\mathrm{w}_2\, \mathrm{X}_2 \leq \mathrm{TAC} < \mathrm{w}_2\, \mathrm{X}_2 + [(1-\mathrm{j}_2)/\mathrm{j}_2]\, \mathrm{w}_2\, \mathrm{X}_2$.

Result 2: In a fishery consisting of two fishermen characterized by their fishing technologies such that fisherman 1 has perfect fishing selectivity and fisherman 2 has imperfect fishing selectivity, initial allocation of quotas does matter to achieve the MSY and hence sustainable fisheries.

So far we studied two cases on the implementation of MSY. The last case investigates the optimal harvesting policy for fishing environments with two fishermen having imperfect fishing selectivity.

Case 3: *Suppose that both fishermen have imperfect fishing selectivity* $(j_1 < 1, and\ j_2 < 1)$.

Case 3.1: If the optimal fishing mortalities of the young mature and old mature fish are found as $0 < f_1 < 1$ and $f_2 = 1$ and TAC is such that $w_2\, X_2 \leq TAC = f_1\, w_1\, X_1 + w_2\, X_2 < \min\{w_2\, X_2 + [(1-j_1)/j_1] w_2\, X_2,\ w_2\, X_2 + [(1-j_2)/j_2] w_2\, X_2\}$, then MSY will not be achieved since the old mature fish population will not be fully harvested due to the fact that

TAC is set below the cut-off levels. The weight of the old mature fish catch will be equal to $j_1\,\alpha_1\,TAC + j_2\,\alpha_2\,TAC = h_2^1\ w_2\,X_2\ + h_2^2\ w_2\,X_2 < w_2\,X_2$. As a result, Case 3.1 is the only case in which it is not possible to achieve MSY harvesting outcomes among the cases represented in this study.

Case 3.2: The optimal fishing mortalities of the young mature and old mature fish are found as $0 < f_1 < 1$, $f_2 = 1$, and TAC is determined at a level satisfying the condition: $w_2\,X_2 + [(1-j_1)\,/\,j_1]\ w_2\,X_2\ \leq TAC < w_2\,X_2 + [(1-j_2)/j_2]\ w_2\,X_2$. Under these conditions, MSY is achieved if all quotas are assigned to fisherman 1. In such a case, fisherman 1 catches $w_2\,X_2$ tonnes of old mature fish and $f_1\,w_1\,X_1$ tonnes of young mature fish, where $f_1\,w_1\,X_1 = \varepsilon +\ [(1-j_1)\,/\,j_1]\ w_2\,X_2$ and ε is equal to the weight of young mature fish caught by fisherman 1 to fulfill his remaining quotas after the old mature fish population is fully harvested $(0 \leq \varepsilon < \{[(1-j_2)/j_2]\ w_2\,X_2 - [(1-j_1)\,/\,j_1]\ w_2\,X_2\})$. However, at the other corner solution, $\alpha_2 = 1$ and $\alpha_1 = 0$, fisherman 2 fulfills his quota before the old mature fish population is fully harvested. Hence, MSY will not be achieved. As a result, it can be stated that there is a lower bound for α_1. Let's check for the lower bound of quota share to be assigned to fisherman 1.

We look for interior solutions. To have an interior solution, the old mature fish population should be fully harvested. As we discussed before, the larger α_1 results in a higher weight of old mature fish catch (in case 3.2, where $j_1 >\ j_2$) until all of the old mature fish are captured.[5] Hence, there is a lower bound for α_1 ; $\underline{\alpha}_1$. Below this level, total catch of the old mature fish will be less than $w_2\,X_2$. In this case, there is no waste of quota since both fishermen fulfill their remaining quotas by capturing young mature fish after the old mature fish population is fully caught. For any given TAC level, there is $\alpha \in [0,1)$ such that $TAC = f_1\,w_1\,X_1 +\ w_2\,X_2 = w_2\,X_2 + \alpha\,w_2\,X_2[(1-j_1)/j_1] + (1-\alpha\,)w_2\,X_2[(1-j_2)/j_2]$. This

[5] The results for the other case in which $j_2 >\ j_1$ are also similar, and hence we do not consider that case.

implies that $\alpha = \frac{(w_2 X_2 - j_2 TAC) j_1}{(j_1 - j_2) w_2 X_2}$. For any feasible allocation of fishing rights, we can find $\alpha' \in (0,1]$ such that $h_2^1 w_2 X_2 = \alpha' w_2 X_2$, $h_2^2 w_2 X_2 = (1-\alpha') w_2 X_2$, $b_1^1 w_1 X_1 = \alpha' w_2 X_2[(1-j_1)/j_1] + \theta$, $b_1^2 w_1 X_1 = f_1 w_1 X_1 - b_1^2 w_1 X_1 = (1-\alpha') w_2 X_2[(1-j_2)/j_2] + \psi$ where $\theta, \psi \geq 0$, and hence $\alpha' \geq \alpha$. It is easy to see that $\alpha' = \alpha$ at $\underline{\alpha}_1$. This is to say that $\theta = \psi = 0$. Therefore, $j_1 \underline{\alpha}_1 TAC = \alpha w_2 X_2$. Then, $\underline{\alpha}_1 = \alpha w_2 X_2 / j_1 TAC = \frac{[w_2 X_2 - j_2 TAC]}{(j_1 - j_2) TAC}$ to achieve MSY harvesting conditions.

Case 3.3: Suppose that optimal fishing mortalities of the young mature and old mature fish are found as $0 < f_1 < 1$ and $f_2 = 1$. Moreover, TAC is determined at a level such that $TAC \geq \max\{w_2 X_2 + [(1-j_1)/j_1] w_2 X_2, w_2 X_2 + [(1-j_2)/j_2] w_2 X_2\}$. Total catch of old mature fish is always equal to $w_2 X_2$ since TAC is set higher than or equal to the maximum of two cut-off levels. Furthermore, both fishermen catch young mature fish to fulfill their quotas after the old mature fish population is fully harvested since $f_1 w_1 X_1 \geq [(1-j_1)/j_1] w_2 X_2$ and $f_1 w_1 X_1 \geq [(1-j_2)/j_2] w_2 X_2$. As a result, the total catch of the young mature fish is equal to $f_1 w_1 X_1$. Therefore, MSY is achieved independently of the quota allocation mechanism. Optimal allocation of quotas at different fishing mortalities for Case 3 is summarized in Table 6.

Table 6 Quota allocation mechanism for Case 3

<table>
<tr><td></td><td>The discounted biomass conditions</td><td>Fishing mortality rates at MSY</td><td>Optimal allocation of quotas at MSY</td></tr>
<tr><td rowspan="6">Case 3

$0.5 < j_1 < 1,$
$0.5 < j_2 < 1$</td><td colspan="3">i. $w_2\,X_2 \leq TAC = f_1\,w_1\,X_1 + w_2\,X_2 <$
$\min\{w_2\,X_2 + [(1-j_1)/j_1]\,w_2\,X_2, w_2\,X_2 + [(1-j_2)/j_2]w_2\,X_2\}$</td></tr>
<tr><td>$\mu = w_1/s_1 < w_2/s_2$</td><td>$0 < f_1 < 1, f_2 = 1$</td><td>MSY is not achieved</td></tr>
<tr><td colspan="3">ii. $w_2\,X_2 + w_2\,X_2\,[(1-j_1)\,/\,j_1] \leq TAC = f_1\,w_1\,X_1 + w_2\,X_2$
$TAC = f_1\,w_1\,X_1 + w_2\,X_2 < w_2\,X_2 + [(1-j_2)/j_2]\,w_2\,X_2$</td></tr>
<tr><td>$\mu = w_1/s_1 < w_2/s_2$</td><td>$0 < f_1 < 1$
$f_2 = 1$</td><td>$\frac{[w_2\,X_2 - j_2 TAC]}{(j_1 - j_2)TAC} \leq \alpha_1 \leq 1$
$\alpha_2 = 1 - \alpha_1$</td></tr>
<tr><td colspan="3">iii. $TAC = f_1\,w_1\,X_1 + w_2\,X_2 \geq \max\{w_2\,X_2 +$
$[(1-j_1)/j_1]\,w_2\,X_2, w_2\,X_2 + [(1-j_2)/j_2]\,w_2\,X_2\}$</td></tr>
<tr><td>$\mu = w_1/s_1 < w_2/s_2$</td><td>$0 < f_1 < 1,$
$f_2 = 1$</td><td>$\{\alpha \mid \alpha_i \in [0,1] \wedge \alpha_1 + \alpha_2 = 1\}$</td></tr>
</table>

We can now state the following result for this case using the findings in the above table.

Result 3: In a fishery consisting of two fishermen characterized by their fishing technologies such that both fishermen have imperfect fishing selectivity, allocation of quotas does matter in achievement of MSY depending on the age distribution of the fish population and fishing technology.

5 Numerical Illustration

In this part of the paper, a simple numerical example is given to clarify the implementation of the quota allocation mechanism described in previous sections. Specifically, we exemplify the optimal quota levels derived in Case 2. Besides the fact that it is one of the most complex cases carried out in this study, it also gives the opportunity to compare suggested quota shares of two fishermen who have different fishing structures. We generated the random data set below, which is not related to any particular fisheries.

Table 7 Parameters of a random fishery with a single fish stock

Parameter	Description	Given Values
w_1	Weight for the young mature fish	3.0 (kg/per fish)
w_2	Weight for the old mature fish	5.0 (kg/per fish)
f_1	Fishing mortality for the young mature fish (at MSY)	0.1
f_2	Fishing mortality for the old mature fish (at MSY)	1
X_1	Total population of the young mature fish	100,000
X_2	Total population of the old mature fish	45,000
q_1^2	Catchability coefficient (fisherman 1)	0.04 (1/effort)
q_1^1	Bycatch coefficient (fisherman 1)	0 (1/effort)
q_2^2	Catchability coefficient (fisherman 2)	0.04 (1/effort)
q_2^1	Bycatch coefficient (fisherman 2)	0.01 (1/effort)

It should be noted that in this section we do not compute MSY harvesting outcomes. We assume that optimal fishing mortalities at MSY are given as above. Since total population and average weight per fish values are given, we can simply calculate the total biomass for each age group of fish. Furthermore, by using Eq. 12, we calculate the TAC (=MSY) level under fix fishing mortalities.

$$MSY = TAC = f_1\, w_1\, X_1 + f_2\, w_2\, X_2 = 255 \text{ tonnes.}$$

As being one of the key parameters of the described mechanism, fishing technologies are calculated as follows:

$$j_1 = q_1^2/q_1^2 + q_1^1 = 0.04/(0.04 + 0) = 1,$$

$$j_2 = q_2^2/q_2^2 + q_2^1 = 0.04/(0.04 + 0.01) = 0.8.$$

Given the fact that the only fisherman who has imperfect fishing selectivity is fisherman 2, it can be deduced that there is only one cut-off level for the TAC that can be written as:

$$w_2\, X_2 + [(1 - j_2)/j_2]\, w_2\, X_2 = 281.25 \text{ tonnes.}$$

We find that MSY is higher than the total weight of the young mature fish ($w_2\, X_2$) and less than the cut-off level of $w_2\, X_2 + [(1 - j_2)/j_2]\, w_2\, X_2$. Hence MSY=TAC is at a level satisfying the condition of $2\mathrm{w}_2\, \mathrm{X}_2/(1 + \mathrm{j}_2) \leq TAC < w_2\, X_2 + [(1 - j_2)/j_2]\, w_2\, X_2$.[6] The optimal solution for this case was found as the following (Case 2.2.a):

[6] Note that the critical fishing mortality rate for the young mature fish is $f^* \cong 0.08 < f_1 = 0.1$.

$$\frac{[w_2\, X_2 - j_2\, TAC]}{(1-j_2)TAC} \leq \alpha_1 \leq \frac{w_2\, X_2}{(1+j_2)TAC}$$

$$\alpha_2 = 1 - \alpha_1.$$

The optimal quota shares computed by the equations above are, $\frac{21}{51}(\cong 0.4118) \leq \alpha_1 \leq \frac{225}{459}(\cong 0.4902)$ and $\frac{30}{51}(\cong 0.5098) \leq \alpha_2 \leq \frac{234}{459}(\cong 0.5882)$. There are two general results to be deduced from the stated quota shares. Firstly, provided that $j_2 < 1$, a higher level of fishing technology of fisherman 2 will result in a higher quota level for fisherman 2. Secondly, suppose that as a result of the maximization problem f_1 is found at a higher rate resulting in a TAC level which is higher than the calculated one but still satisfying the condition of $2w_2\, X_2/(1+j_2) \leq TAC < w_2\, X_2 + [(1-j_2)/j_2]\, w_2\, X_2$. In such a case, optimal quota share for fisherman 2 will be higher. As a result, we can state that the optimal quota levels depend on both fishing technologies and optimal fishing mortality levels.

6 Conclusion

In the reform process of the CFP, the EU is seeking for an economically and socially viable, well-designed management system for EU fisheries. In this regard, the EU promotes measures for achieving and sustaining MSY. This paper examines the problem of designing quota allocation mechanisms or management systems to implement MSY fishing levels. We show that not only biological limitations due to structure of the fish population but also composition of fisheries and different fishing technologies should be taken into consideration in determination of maximum catch limits (or property rights). Furthermore, the analysis shows that the optimal solution for allocation of quotas is highly dependent on MSY (=TAC) level. Thus, one of the important policy implications of our analysis is that fishing technologies and

TAC levels should be analyzed together while distributing fishing quotas (or assigning property rights) to fishing agents.

In the model part, firstly the population model and the quota allocation mechanism are described and then fishing mortalities at MSY are derived. Afterwards, optimal quota levels are calculated by using the proposed mechanism. Under the condition that fishermen fulfill their remaining quotas through capturing less revenue-generating fish after the targeted fish population is fully caught, the fix ratio of catch of targeted fish to bycatch would no longer be valid. Optimal allocations of quotas are determined under the consideration of this non-satiated behavior of fishermen. Accordingly, we determine technology-based optimal individual quota levels at particular MSY. Furthermore, this estimation enables us to prevent high grading under a well-functioning control mechanism for landings of fishermen since we can estimate catch composition of fishermen under specified quota levels.

In the EU, TACs are determined at the Union level and distributed to the EU countries based on the principle of 'relative stability'.[7] Member States use different management systems to allocate these assigned national quotas to domestic fishermen. The initial allocation is usually determined by grandfathering, a proportional rule based on historical catches of existing fishers. It is also possible to use auctions to determine the initial allocation of national quotas. We show that allocating the quotas according to this history depended proportional distribution rule or auctioning may not provide economically and biologically viable solutions to achieve MSY harvesting condition since the allocation rule should depend on the age distribution of the fish population and fishing technology composition of domestic fishermen. Therefore, one of the main suggestions of this paper is that the technological structure of fishing industry and the structure of fish population should be considered in the process of distributing national quotas so as to achieve MSY.

[7] See the European Commission fisheries webpage, http://ec.europa.eu/fisheries/index_en.htm, for more on EU fishing rules.

In this study, we concentrate on a simple model to investigate the implementation problem of MSY under an individual quota system. It is left for further research to improve the analysis in this paper by considering fishing and management costs and strategic response of fishermen to different management systems.

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